Milne's predictor - Corrector method

Introduction :

Predictor-Corrector methods are methods which require function values at $x_n, x_{n-1}, x_{n-2}, x_{n-3}$ for the computation of the function value at x_{n+1}. A predictor is used to find the value of y at x_{n+1} and then a Corrector formula is used to improve the value of y_{n+1}.

Milne's predictor formula

$$y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

and the error $= \frac{14h^5}{45} y^{(v)}(\xi)$ where $x_{n-3} < \xi < x_{n+1}$.

Milne's Corrector formula

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

and the error $= \frac{-h^5}{90} y^{(v)}(\xi)$ where $x_{n-1} < \xi < x_{n+1}$.

PROBLEMS :

1. Using Milne's method, Compute $y(0.8)$ given that $\frac{dy}{dx} = 1+y^2$, $y(0)=1$, $y(0.2) = 0.2027$, $y(0.4)=0.4228$ and $y(0.6) = 0.6841$.

Solution :-

we have the following table of values

x	y	$y' = 1+y^2$
0	0	1.0
0.2	0.2027	1.0411
0.4	0.4228	1.1787
0.6	0.6841	1.4681

$\therefore$ $x_0 = 0$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$

$y_0 = 0$, $y_1 = 0.2027$, $y_2 = 0.4228$, $y_3 = 0.6841$

$y_0' = 1$, $y_1' = 1.0411$, $y_2' = 1.1787$, $y_3' = 1.4681$

To find $y(0.8)$:

$x_4 = 0.8$. Here $h = 0.2$

By Milne's predictor formula,

$$y_{4,P} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

$$= 0 + \frac{0.8}{3}[2(1.0411) - 1.1787 + 2(1.4681)]$$

$$\therefore \boxed{y_{4,P} = 1.0289}$$

$$y_4' = 1 + (1.0289)^2 = 2.0480$$

By Milne's corrector formula,

$$y_{4,C} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 0.4228 + \frac{0.2}{3}[1.1787 + 4(1.4681) + 2.0480]$$

$$= 1.0294$$

$$\therefore \boxed{y(0.8) = 1.0294}$$

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2. Use Milne's predictor - Corrector formula to find $y(0.4)$, given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$, $y(0)=1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$ and $y(0.3) = 1.21$.

Solution :

Given $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$, $x_4 = 0.4$.

$y_0 = 1$, $y_1 = 1.06$, $y_2 = 1.12$, $y_3 = 1.21$, $h = 0.1$.

Given: $y' = \frac{1}{2}[1+x^2]y^2$.

$$y_0' = \frac{1}{2}[1+x_0^2]y_0^2 = \frac{1}{2}(1+0)(1) = \frac{1}{2} = 0.5$$

$$y_1' = \frac{1}{2}[1+x_1^2]y_1^2 = \frac{1}{2}[1+(0.1)^2][1.06]^2$$

$$= \frac{1}{2}(1.01)(1.1236) = 0.5674$$

$$y_2' = \frac{1}{2}[1+x_2^2]y_2^2 = \frac{1}{2}[1+(0.2)^2][1.12]^2$$

$$= \frac{1}{2}[1.04][1.2544] = 0.6522$$

$$y_3' = \frac{1}{2}[1+x_3^2]y_3^2 = \frac{1}{2}[1+(0.3)^2][1.21]^2$$

$$= \frac{1}{2}[1.09][1.4641] = 0.7979$$

By Milne's method

$$y_{4,P} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

$$= 1 + \frac{4(0.1)}{3}[2(0.5764) - (0.6522) + 2(0.7979)]$$

$$= 1.2771$$

$$y_4' = \frac{1}{2}[1+x_4^2]y_4^2$$

$$= \frac{1}{2}[1+(0.4)^2](1.2771)^2$$

$$= \frac{1}{2}(1.16)(1.631) = 0.9460$$

By Corrector method

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 1.12 + \frac{0.1}{3}[0.6522 + 4(0.7979) + 0.9460]$$

$$\boxed{y_{4,c} = 1.2797}$$

(3). Given $y' = \frac{1}{x+y}$, $y(0)=2$, $y(0.2) = 2.0988$, $y(0.4) = 2.1755$, $y(0.6) = 2.2493$, $y(0.8)$ by Milne's predictor corrector method.

Ans

$$y_{4,P} = 2.3162$$

$$y_{4,C} = 2.3164.$$